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Equation of State of Newborn Neutron Star Matter With Untrapped Neutrinos

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The equation of state of the newborn neutron star matter with untrapped neutrinos is calculated with the AV_{18} potential along isentropic paths. The same calculations are done with the AV_{14} potential for the sake of comparison. Temperature–density correlation, proton fraction, adiabatic index, and the velocity of sound are also obtained at different entropies. It is shown that the proton fraction (adiabatic index) increases (decreases) by increasing entropy. We have shown that our calculated equations of state obey the causality condition. The results are compared with those of others in the literature.

1. INTRODUCTION

Neutron stars are formed in the gravitational collapse of supernovae (Shapiro and Teukolsky, 1983). Newborn neutron stars differ from ordinary neutron stars, in the sense that the matter inside them has nearly constant entropy per nucleon of the order 1–2 in the units of the Boltzmann constant (k_B) (Bethe *et al.*, 1979; Burrows and Lattimer, 1986; Keil and Janka, 1995; Pons *et al.*, 1999; Sumiyoshi *et al.*, 1995). In the case of untrapped neutrinos, we can consider a matter consisting of neutrons, protons, electrons, and muons under conditions of charge neutrality and beta equilibrium (Burrows and Lattimer, 1986; Keil and Janka, 1995; Pons *et al.*, 1999; Sumiyoshi *et al.*, 1995).

The properties of the neutron star matter, especially its equation of state, have a crucial role for studying the structure and evolution and determining the mass of neutron stars (Bombaci, 1996; Lattimer *et al.*, 1991; Prakash, 1994; Prakash *et al.*, 1988).

In recent years, the equation of state of the neutron star matter is calculated by various many-body methods. The results of these calculations show large

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differences, since these are based on various models of the nucleon–nucleon potentials that are not phase-shift equivalent (Engvik *et al.*, 1997). On the contrary, by using the modern potentials, such as the new Argonne potential (AV_{18} potential) (Wiringa *et al.*, 1995), these differences become small (Engvik *et al.*, 1997). This feature is very important for a more precise mass determination (Bombaci, 1996; Prakash *et al.*, 1988).

Recently, we have used the lowest order constrained variational (LOCV) method for investigating the properties of nuclear matter at zero (Bordbar and Modarres, 1997, 1998) and finite (Bordbar, in press; Modarres and Bordbar, 1998) temperatures. More recently, we have calculated the equation of state of cold neutron star matter and some of its properties (Bordbar and Riazi, submitted).

In the present paper, we introduce the equation of state of newborn neutron star matter and some of its properties in the case of untrapped neutrinos, using LOCV method. In our calculations, we employ the AV_{18} potential (Wiringa *et al.*, 1995), together with the AV_{14} potential (Wiringa *et al.*, 1984) for the sake of comparison.

2. LOCV METHOD AT $\mathcal{T} \neq 0$

By using variational method, we can write the wave function of a system of A interacting particles as

$$\psi = F\phi, \tag{1}$$

where the A-particle correlation function is taken to be

$$F = \mathcal{S} \prod_{i>j} f(ij), \tag{2}$$

and ϕ is a Slater determinant of the single-particle wave functions. In the aforementioned equation, $\mathcal{S}$ is the symmetrizing operator and the two-body correlation operators $f(ij)$ are as follows

$$f(ij) = \sum_{\alpha,p=1}^{3} f_\alpha^p(ij) O_\alpha^p(ij), \tag{3}$$

where $\alpha = \{J, L, S, T, T_z\}$ and

$$O_\alpha^{p=1,3} = 1,\ \left(\frac{2}{3} + \frac{1}{6}S_{12}\right),\ \left(\frac{1}{3} - \frac{1}{6}S_{12}\right), \tag{4}$$

where S_{12} is the tensor operator (Wiringa *et al.*, 1995).

Now, we consider up to the two-body term in the cluster expansion for the energy functional,

$$E([f]) = \frac{1}{A}\frac{\langle\psi \mid H \mid \psi\rangle}{\langle\psi \mid \psi\rangle} = E_1 + E_2. \tag{5}$$

For nucleonic matter at finite temperature ($\mathcal{T} \neq 0$), E_1 is the Fermi-gas kinetic energy expression,

$$E_1 = \sum_{i=n,p} \sum_k \frac{\hbar^2 k^2}{2m_i} n_i(k, \mathcal{T}, \rho_i), \tag{6}$$

where $n_i(k, \mathcal{T}, \rho_i)$ is the Fermi–Dirac distribution function,

$$n_i(k, \mathcal{T}, \rho_i) = \frac{1}{e^{\beta[\epsilon_i(\kappa, \mathcal{T}, \rho_i) - \mu_i(\mathcal{T}, \rho_i)]} + 1}. \tag{7}$$

In the aforementioned equation $\beta = \frac{1}{k_B \mathcal{T}}$ and μ_i are the chemical potentials, ρ_i are the nucleonic number densities, and

$$\epsilon_i(k, \mathcal{T}, \rho_i) = \frac{\hbar^2 k^2}{2m_i^*(\mathcal{T}, \rho_i)} \tag{8}$$

are the single particle energies, associated with the protons and neutrons. The m_i^* are the effective masses.

The two-body energy, E_2, is

$$E_2 = \frac{1}{2A} \sum_{ij} \langle ij \mid \mathcal{V}(12) \mid ij - ji \rangle, \tag{9}$$

where

$$\mathcal{V}(12) = -\frac{\hbar^2}{2m} \left[f(12), \left[\nabla_{12}^2, f(12) \right] \right] + f(12) V(12) f(12). \tag{10}$$

The general form of two-body nucleon–nucleon interaction, $V(12)$, is (Wiringa *et al.*, 1995)

$$V(12) = \sum_{p=1}^{18} V^p(r_{12}) O_{12}^p, \tag{11}$$

where

$$\begin{aligned} O_{12}^{p=1-18} = {} & 1, \sigma_1 \cdot \sigma_2, \tau_1 \cdot \tau_2, (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2), S_{12}, S_{12}(\tau_1 \cdot \tau_2), \\ & \mathbf{L} \cdot \mathbf{S}, \mathbf{L} \cdot \mathbf{S}(\tau_1 \cdot \tau_2), \mathbf{L}^2, \mathbf{L}^2(\sigma_1 \cdot \sigma_2), \mathbf{L}^2(\tau_1 \cdot \tau_2), \\ & \mathbf{L}^2(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2), (\mathbf{L} \cdot \mathbf{S})^2, (\mathbf{L} \cdot \mathbf{S})^2(\tau_1 \cdot \tau_2), \\ & \mathbf{T}_{12}, (\sigma_1 \cdot \sigma_2)\mathbf{T}_{12}, S_{12}\mathbf{T}_{12}, (\tau_{z1} \cdot \tau_{z2}). \end{aligned} \tag{12}$$

Here, T_{12} is the isotensor operator (Wiringa *et al.*, 1995).

Now, as in our previous calculations, we minimize the two-body energy with respect to the variations in the correlation functions f_α^p but subject to the normalization constraint (Bordbar, in press; Bordbar and Modarres, 1997, 1998; Modarres and Bordbar, 1998). Each correlation function f_α^p is required to heal to the Pauli function h_{T_z},

$$h_{T_z} = \begin{cases} \left[1 - \frac{1}{\nu}\left(\frac{\Gamma_i(r)}{\rho}\right)^2\right]^{-\frac{1}{2}}, & T_z = \pm 1 \\ 1, & T_z = 0, \end{cases} \tag{13}$$

where

$$\Gamma_i(r) = \frac{2\nu}{(2\pi)^2} \int_0^\infty n_i(k, \mathcal{T}, \rho_i) J_0(kr) k^2 dk, \tag{14}$$

and $\nu = 2$. The total nucleonic number density, ρ, is

$$\rho = \rho_{\mathrm{p}} + \rho_{\mathrm{n}}. \tag{15}$$

By minimizing the two-body energy, we get a set of Euler–Lagrange differential equations similar to those described in our previous works (Bordbar and Modarres, 1998). The procedure for the calculation of energy has been fully discussed in Bordbar and Modarres (1998).

3. EQUATION OF STATE

Now, we consider the newborn neutron star matter in the case of untrapped neutrinos, which has nearly constant entropy per nucleon, $s \sim 1$–2. This matter is an uncharged composition of neutrons, protons, electrons, and muons, that is in beta equilibrium.

The contribution from the energy of leptons (electrons and muons), which should be added to Eq. (5), is as follows

$$E_{\mathrm{L}} = \sum_{i=e,\mu} \sum_k n_i(k, \mathcal{T}, \rho_i)[(m_i c^2)^2 + \hbar^2 c^2 k^2]^{1/2}. \tag{16}$$

The conditions of charge neutrality and beta equilibrium impose the following constraints on the calculation of the energy of neutron star matter,

$$\rho_{\mathrm{p}} = \rho_{\mathrm{e}} + \rho_\mu \tag{17}$$

and

$$\mu_{\mathrm{n}} - \mu_{\mathrm{p}} = \mu_{\mathrm{e}} = \mu_\mu. \tag{18}$$

The equation of state of newborn neutron star matter, $P(\rho, s)$, can be simply obtained using

$$P(\rho, s) = \rho^2 \frac{\partial E(\rho, s)}{\partial \rho}, \tag{19}$$

where s is the entropy per nucleon. In Fig. 1, we have presented the pressure of newborn neutron star matter as a function of total number density (ρ) with the AV_{18} and AV_{14} potentials at different entropies ($s = 1.0, 2.0$). We have also presented the results of our calculations for the cold neutron star matter ($s = 0.0$) (Bordbar and Riazi, submitted). We see that for all values of entropy, the equation of state with the AV_{18} potential is stiffer than those with the AV_{14} potential. We also see that with increasing density, the differences between the equations of state at different

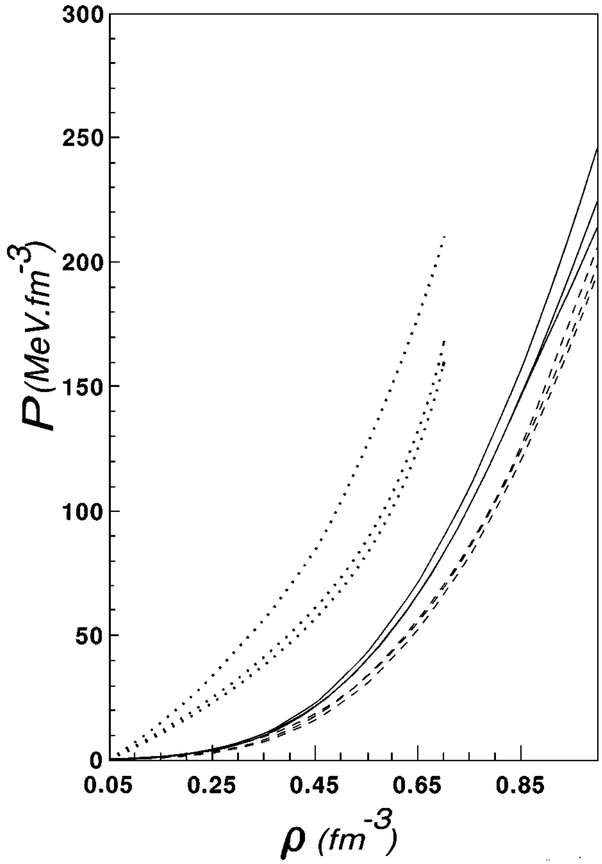

Fig. 1. The equation of state of newborn neutron star matter at $s = 2.0$ (upper curves) and 1.0 (middle curves) and cold neutron star matter (lower curves) with the AV_{18} (full curves) and AV_{14} (dashed curves) potentials. The results of Strobel *et al.* (1999) (dotted curves) are given for comparison.

entropies become more appreciable. In this figure, we have also shown the results of Strobel *et al.* (1999) for comparison. We can see that the equations of state of Strobel *et al.* (1999) are much harder than those of ours. It can also be seen that the differences between the results of Strobel *et al.* (1999) at different entropies are more appreciable relative to our results. This is because we do not vary the effective masses (m_i^*) and choose $m_i^* = m_i$ in our calculations, since in the case of constant entropy, we have found that the internal energy does not change with these parameters (Modarres and Bordbar, 1998).

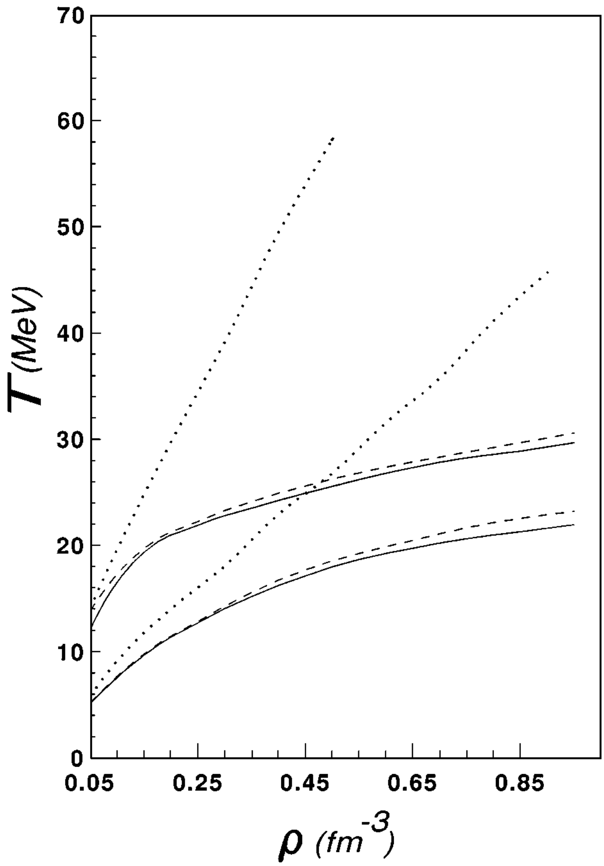

Fig. 2. Temperature versus total number density for newborn neutron star matter at $s = 2.0$ (upper curves) and 1.0 (middle curves) with the AV_{18} (full curves) and AV_{14} (dashed curves) potentials. The results of Strobel *et al.* (1999) (dotted curves) are given for comparison.

The temperature of newborn neutron star matter as a function of total number density with the AV_{18} and AV_{14} potentials at $s = 1.0$ and 2.0 is given in Fig. 2. We see that the temperature increases with increasing entropy. Also, the calculated temperatures with the AV_{18} potential are nearly identical with those of AV_{14} potential, especially at low densities. In Fig. 2, we have also shown the results of Strobel *et al.* (1999). The large differences between our results and the results of Strobel *et al.* (1999) are due to the choice of the effective masses, as described in the previous paragraph.

In Fig. 3, we have plotted the proton fraction of the newborn neutron star matter as a function of total number density with the AV_{18} and AV_{14} potentials

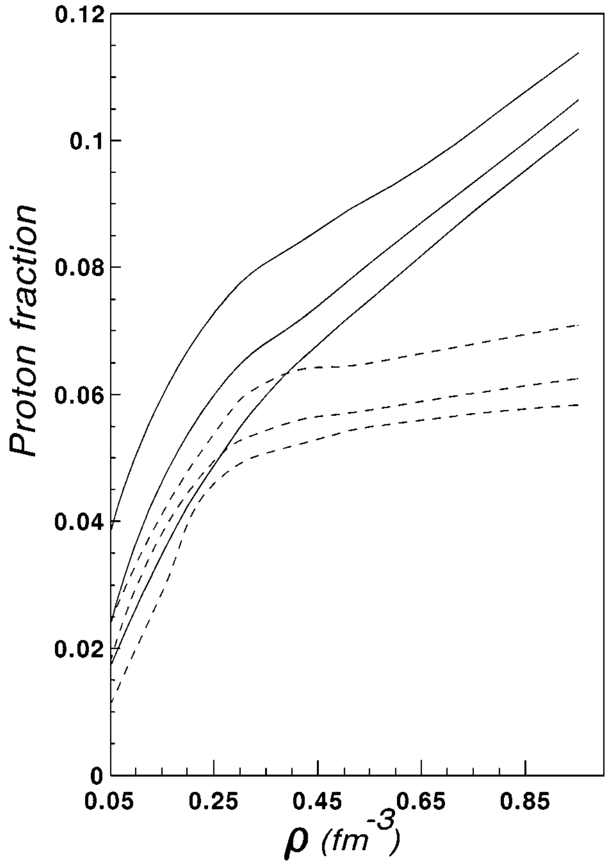

Fig. 3. Proton fraction versus total number density for newborn neutron star matter at $s = 2.0$ (upper curves) and 1.0 (middle curves) and cold neutron star matter (lower curves) with the AV_{18} (full curves) and AV_{14} (dashed curves) potentials.

at $s = 1.0$ and 2.0 as well as for cold neutron star matter ($s = 0.0$) (Bordbar and Riazi, submitted). It is seen that the proton fraction increases by increasing entropy. This result has an important implication in the investigation of the cooling of neutron stars (Lattimer *et al.*, 1991; Prakash, 1994). Also, the calculations with the AV_{18} potential show a higher proton fraction than those with the AV_{14} potential, especially at high densities.

The adiabatic index, γ, is defined by

$$\gamma = \left(\frac{\partial ln P}{\partial ln \rho}\right)_s. \tag{20}$$

This parameter is of crucial importance for the core bounce of the collapsing stars. It is well known that as the density approaches to nuclear matter density, γ rises above $4/3$ (a value that corresponds to relativistic leptons). This means that the collapse comes rapidly to a halt, and is reversed into a bounce. Furthermore, the stability of a star depends on the value of γ in the core (Shapiro and Teukolsky, 1983). In Fig. 4, we have shown the results of our calculations for the adiabatic index (γ) with the AV_{18} and AV_{14} potentials at different entropies ($s = 0.0, 1.0, 2.0$). It can be seen that $\gamma > 4/3$ even at low densities and becomes even more at high densities. We can also see that γ decreases with increasing

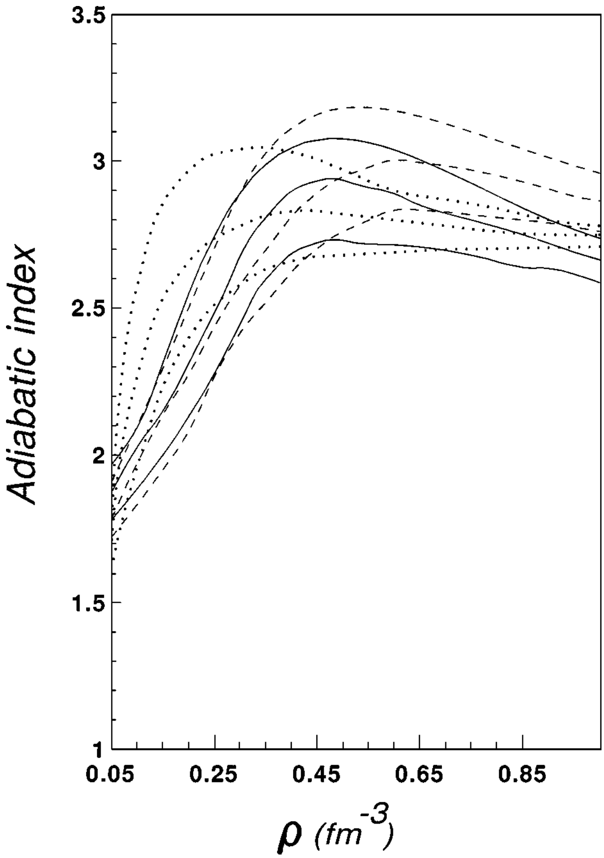

Fig. 4. Adiabatic index versus total number density for newborn neutron star matter at $s = 2.0$ (lower curves) and 1.0 (middle curves) and cold neutron star matter (upper curves) with the AV_{18} (full curves) and AV_{14} (dashed curves) potentials. The results of Strobel *et al.* (1999) (dotted curves) are given for comparison.

entropy. In this figure, we have also presented the results of Strobel *et al.* (1999). There is an overall agreement between our results and those of Strobel *et al.* (1999).

In order to check the causality condition of equations of state calculated in this paper, we calculate the velocity of sound, v_s, according to

$$\frac{v_s}{c} = \left(\frac{dP}{d\varepsilon}\right)^{1/2}, \tag{21}$$

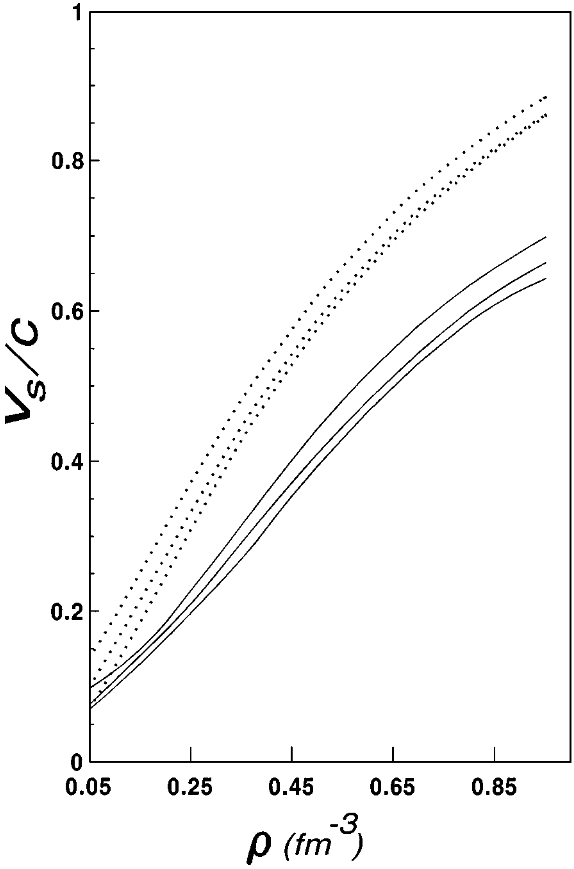

Fig. 5. Velocity of sound (in the units of the velocity of light in the vacuum) versus total number density at $s = 2.0$ (upper curves), 1.0 (middle curves), and 0.0 (lower curves) with the AV_{18} (full curves) potential. The results of Strobel *et al.* (1999) (dotted curves) are given for comparison.

where ε is the mass–energy density,

$$\varepsilon = \rho(E + mc^2), \tag{22}$$

m is the nucleon mass, and c is the velocity of light in the vacuum. In Fig. 5, the velocity of sound (in the units of c) is shown with the AV_{18} potential at $s = 0.0$, 1.0, and 2.0. The velocity of sound with the AV_{14} potential (which is not shown in this figure) is nearly identical to that of the AV_{18} potential. It is seen that the velocity of sound increases with both increasing entropy and density, but it is always lower than the velocity of light in vacuum (c). Therefore, all equations of state calculated in this paper obey the causality condition. In Fig. 5, we have also presented the results of Strobel *et al.* (1999). There is a considerable difference between the two calculations, which is caused by the differences in the equations of state, as discussed earlier.

4. SUMMARY AND CONCLUSION

The equation of state of the newborn neutron star matter in the case of untrapped neutrinos was presented. Our calculations were based on the LOCV method. The nucleon–nucleon potentials used in these calculations were AV_{18} and AV_{14}. The following properties were also calculated and compared to the results of Strobel *et al.* (1999):

- temperature–density relation,
- proton fraction as a function of total number density,
- adiabatic index as a function of total number density, and
- the velocity of sound as a function of total number density.

Our results indicated that the entropy affects the equation of state, especially at high densities. The temperature and proton fraction were found to increase with the entropy. The adiabatic index was found to be greater than $4/3$ at all densities (and entropies). It is interesting to note that this parameter becomes fairly constant at densities beyond $\sim 0.6\ \mathrm{fm}^{-3}$. Finally, the causality condition was shown to hold for all of the results presented in this paper.

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